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ASYMPTOTIC OF SOLUTION OF PROBLEM OF CONVECTIVE DIFFUSION TO A DROP WITH LARGE PÉCLET NUMBERS AND FINITE REYNOLDS NUMBERS

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A first approximation in the problem of steady-state convective diffusion to a spherical particle in a homogeneous translational flow has been obtained for zero [1] and finite Reynolds numbers [2, 3]. A two-term expansion in the case of Stokes flow around a solid particle is given in [4].

We postulate that the concentration of the substance dissolved in the flow is constant far from the drop, and that it is completely absorbed at the surface. In a spherical system of coordinates connected with the drop, the dimensionless equation of convective diffusion and the boundary conditions have the form (Pe is the Péclet number)

$$v_r \frac{\partial c}{\partial r} + \frac{v_\theta}{r}\frac{\partial c}{\partial \theta} = e^2\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial c}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial c}{\partial\theta}\right)\right\},$$
$$r=1,\quad c=0;\quad r=\infty,\quad c=1, \tag{1}$$
$$v_r = \frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta},\quad v_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r},\quad e^2 = \mathrm{Pe}^{-1} = \frac{D}{aU}.$$

Here the concentration at infinity, the velocity of the oncoming flow U, and the radius of the drop a are taken as the scales of the concentration, the velocity, and the length; the angle θ is reckoned from the direction of the flow at infinity.

For the field of the velocities we use expressions obtained for a drop by the method of joined asymptotic expansions [5]:

$$\psi = \psi_0 + \mathrm{Re}\,\psi_1 \quad (\mathrm{Re} = aU/\nu),$$
$$\psi_0 = \frac{1}{2}(r-1)\left[r - \frac{1}{2}\frac{\beta}{\beta+1}\left(1+\frac{1}{r}\right)\right]\sin^2\theta, \tag{2}$$
$$\psi_1 = \frac{1}{8}\frac{3\beta+2}{\beta+1}\psi_0 - \frac{1}{16}\frac{3\beta+2}{\beta+1}(r-1)\left[r - \frac{1}{2}\frac{\beta}{\beta+1} - \frac{1}{10}\frac{\beta}{(\beta+1)^2}\left(\frac{1}{r} + \frac{5\beta+6}{r^2}\right)\right]\sin^2\theta\cos\theta,$$

where β is the ratio of the viscosities of the drop and the liquid surrounding it; Re is the Reynolds number.

We shall assume that the Péclet number is large (the parameter e is small); we introduce the extended coordinate Y in the diffusional boundary layer and represent the flow function (2) in the form of a series

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$$\psi=\sum_{n=1}^{\infty} e^n Y^n \lambda_n(\theta), \quad \lambda_n(\theta)=\left.\frac{\partial^n\psi}{\partial r^n}\right|_{r=1}, \ Y=\frac{r-1}{e},$$

$$\lambda_1(\theta)=\frac{1}{2(\beta+1)}\left\{1+\frac{1}{8}\operatorname{Re}\gamma(\beta)\,[1-\gamma_1(\beta)\cos\theta]\right\}\sin^2\theta=A(\theta),$$

$$\lambda_2(\theta)=\frac{1}{4}\gamma(\beta)\left\{1+\frac{1}{8}\operatorname{Re}\gamma(\beta)\,[1-\gamma_2(\beta)\cos\theta]\right\}\sin^2\theta,\ \ldots, \tag{3}$$

$$\gamma(\beta)=\frac{3\beta+2}{\beta+1}, \quad \gamma_1(\beta)=\frac{4\beta+5}{5\beta+5}, \quad \gamma_2(\beta)=\frac{5\beta+2}{3\beta+2}\gamma_1(\beta).$$

To bring the starting problem down to the consecutive solution of the equations of thermal conductivity, we introduce the new variables

$$\zeta=A(\theta)\,Y, \quad t=T(\theta)=\int_{\theta}^{\pi} A(\tau)\sin\tau\,d\tau \quad (\theta=T^{-1}(t)) \tag{4}$$

and shall seek the solution in the form

$$c=\sum_{n=0}^{\infty} e^n c_n(\zeta, t). \tag{5}$$

Going over in the starting problem (1) to the variables (4) and using the representation (5), for the consecutive determination of $c_n(\zeta, t)$ we have the following equation and boundary conditions:

$$(\partial/\partial t-\partial^2/\partial\zeta^2)\,c_n=F_n(c_0(\zeta,t),\ldots,c_{n-1}(\zeta,t))=f_n(\zeta,t),$$

$$\zeta=0,\ c_n=0; \ \zeta=\infty, \quad c_n=\begin{cases}1, & n=0,\\ 0, & n\geqslant 1,\end{cases}$$

$$F_0=0, \tag{6}$$

$$F_1(c_0)=-\frac{2}{A^0(t)}\left\{[1-B(t)]\,\zeta\frac{\partial}{\partial t}+\left[1+\frac{1}{2}A^0(t)\right]\frac{d}{dt}\left(\frac{B(t)}{A^0(t)}\right)\zeta^2\frac{\partial}{\partial\zeta}\right\}c_0(\zeta,t),\ \ldots,$$

$$A^0(t)=A(T^{-1}(t)), \ B(t)=\lambda_2(T^{-1}(t))\,[A^0(t)]^{-1},$$

where $F_n(c_0, \ldots, c_{n-1})$ is a coefficient with e^n, determined by substitution of expression (5) into Eq. (3); here we set $c_n = 0$. With such a choice of variables ζ, t (4), the zero term of the representation (5) corresponds to an approximation of the diffusional boundary layer [1].

The formulation of the problem (6) must be supplemented by the condition for the concentration at the point of inflow $\theta^- = \pi$ (the point of degeneration of the diffusional boundary layer) $c(\theta^- = \pi) = 1$ [1]. Therefore, as the initial condition for $c_n(\zeta, t)$ we take

$$c_n(t=0)=\begin{cases}1, & n=0,\\ 0, & n\geqslant 1.\end{cases} \tag{7}$$

The solution of the problem (6), (7) has the form [1, 6]

$$c_n(t,\zeta)=\begin{cases}\dfrac{2}{\sqrt{\pi}}\displaystyle\int_0^{\eta} e^{-\tau^2}d\tau, \quad \eta=\dfrac{\zeta}{2t^{1/2}}, \quad n=0,\\ \displaystyle\int_0^{\infty}\int_0^{t}[G(\zeta,\xi,t-\tau)-G(\zeta,-\xi,t-\tau)]\,f_n(\xi,\tau)\,d\tau d\xi, \quad n\geqslant 1,\end{cases}$$

$$G(\zeta,\xi,t)=\frac{1}{2\sqrt{\pi t}}\exp\left\{-\frac{(\zeta-\xi)^2}{4t}\right\},$$

$$f_1(\xi,\tau)=\frac{2}{A^0(\tau)\sqrt{\pi\tau}}\left\{1+\left(B(\tau)-1+\tau A^0(\tau)\frac{d}{d\tau}\left[\frac{B(\tau)}{A^0(\tau)}\right]\right)\frac{\xi^2}{2\tau}\right\}\times\exp\left(-\frac{\xi^2}{4\tau}\right),\ \ldots$$

The dimensionless differential flow to the drop is determined in the form

$$j(\theta)=\left.\frac{\partial c}{\partial r}\right|_{r=1}=\frac{A(\theta)}{e}\left.\frac{\partial c}{\partial\zeta}\right|_{\zeta=0}=\frac{1}{e}\sum_{n=0}^{\infty} e^n j_n(\theta),$$

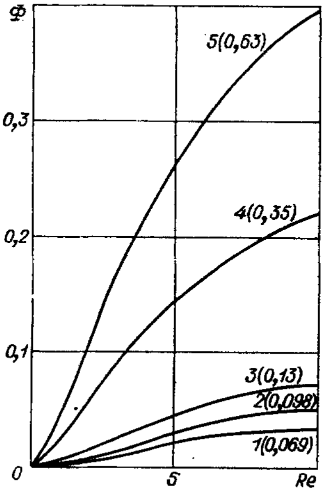

Fig. 1

$$j_n(\theta) = A(\theta)\frac{\partial c_n}{\partial \zeta}\bigg|_{\zeta=0} = A(\theta)\begin{cases} 2\pi^{-1/2}\, t^{-1/2}, & n = 0, \\ \int\limits_0^\infty \int\limits_0^t \Lambda(\xi, t-\tau)\, f_n(\xi,\tau)\, d\xi d\tau, & n \geqslant 1, \end{cases} \qquad (8)$$

$$\Lambda(\xi, t) = \frac{1}{2\sqrt{\pi}}\frac{\xi}{t^{3/2}}\exp\left(-\frac{\xi^2}{4t}\right).$$

For the total diffusional flow to the drop we have

$$I = \int\limits_{\partial\sigma} j d\sigma = \frac{1}{e}\sum_{n=0}^{\infty} e^n I_n,$$

$$I_n = 2\pi\int\limits_0^\pi \sin\theta A(\theta)\frac{\partial c_n}{\partial \zeta}\bigg|_{\zeta=0} d\theta = 2\pi\int\limits_0^{t_0}\frac{\partial c_n}{\partial \zeta}\bigg|_{\zeta=0} dt, \qquad (9)$$

$$t_0 = \frac{2}{3(1+\beta)}\left(1 + \frac{1}{8}\mathrm{Re}\,\gamma(\beta)\right),$$

where the quantity $|\partial c_n/\partial\zeta|_{\zeta=0}$ is defined in expression (8).

The main term of the expansion (9) was obtained in [3]. Transposing the integration limits in expressions (8), (9), and integrating consecutively, first with respect to ξ, and then with respect to t, we obtain the following term of the series (9):

$$\mathrm{Sh}_1(\beta, \mathrm{Re}) = I_1/4\pi = \frac{4}{\pi t_0}\int\limits_0^{t_0}\frac{\sqrt{\tau(t_0-\tau)}}{A^0(\tau)}[1 + B(\tau)]\, d\tau.$$

Using relationships (3), (4), it can be shown that, with Re = 0, the value of $I_1(\beta, 0)$ depends linearly on the parameter β. Therefore, it is convenient to represent the dependence of the Sherwood number on β and Re in the form

$$\mathrm{Sh}_1(\beta, \mathrm{Re}) = 0.825 + 0.620\beta + \Phi(\beta, \mathrm{Re}), \quad \Phi(\beta, 0) = 0. \qquad (10)$$

All the coefficients in the function Φ(β, Re) in expression (10) are obtained numerically. The function Φ(β, Re) is shown in Fig. 1, where the curves 1-5 correspond to the values β = 0, 0.5, 1, 5, 10 [the number in parentheses indicated the value of Φ(β, 100)].

It can be seen that $\mathrm{Sh}_1(\beta, \mathrm{Re})$ rises with a rise in the Reynolds, and that the value of the contribution of Φ(β, Re) to $\mathrm{Sh}_1(\beta, \mathrm{Re})$ with $0 \leq \mathrm{Re} \leq 10$ does not exceed 7%.

Using [3], with an accuracy to $O(\mathrm{Pe}^{-1/2})$ we obtain the following expression for the Sherwood number:

$$\mathrm{Sh}(\beta, \mathrm{Re}) = \mathrm{Pe}^{1/2}(\mathrm{Sh}_0 + \mathrm{Pe}^{1/2}\mathrm{Sh}_1) = 0.460\,\mathrm{Pe}^{1/2}\left[\frac{1}{1+\beta} + \frac{3\beta+2}{(1+\beta)^2}\frac{\mathrm{Re}}{8}\right]^{1/2} + 0.825 + 0.620\beta + \Phi(\beta, \mathrm{Re}). \qquad (11)$$

The region of applicability of formula (11) is limited by the condition $\mathrm{Sh}_0 \gg \mathrm{Pe}^{-1/2}\mathrm{Sh}_1$, i.e., $\beta\mathrm{Pe}^{-1/3} \ll 1$.

We note that a representation for the concentration in the form of the series (9) holds everywhere, with the exception of the neighborhood of the rear critical point $\theta \le O(Pe^{-1/2})$ [1], where the thickness of the diffusional boundary layer $\delta = [j]^{-1}$ becomes infinitely great. Analogously to [7] it can be shown that the contribution of this region to the total diffusional flow to the drop is on the order of $O(Pe^{-1/2})$. Therefore, the calculation of succeeding terms of the series (9) leads to an improvement of formula (11) only in obtaining a solution for the concentration in the region of the rear critical point.

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EQUATIONS OF THERMOHYDROMECHANICS OF A TWO-PHASE POLYDISPERSE MEDIUM WITH PHASE TRANSITIONS HAVING A CONTINUOUS PARTICLE-SIZE DISTRIBUTION

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§1. We consider a heterogeneous mixture of two phases, in which the first phase is the carrier phase, while the second phase is present in the form of individual solid particles of different sizes, direct interaction between which can be neglected. We adopt the hypothesis of quasihomogeneity [1-3]: the distances at which the parameters of the flow vary significantly are much greater than the sizes of the particles themselves and the distances between them. At each point of the volume occupied by the liquid we can introduce the volumetric contents of the phases α_i and the mean densities ρ_i; here

$$\rho = \rho_1 + \rho_2,\ \alpha_1 + \alpha_2 = 1,\ \alpha_i \geqslant 0,\ \rho_i = \rho_i^0 \alpha_i,$$

where the subscript 1 relates to the carrier phase, and 2 to the whole disperse phase; ρ_i is the density of the i-th component of the mixture. The dispersivity of the second phase is characterized by the function f(r), so that f(r)dr is the number of particles in unit volume of the mixture, whose dimensions (volumes) lie within the limits from r to r + dr. The density of the second phase is continuously distributed in the segment [0, R], where R is the dimension (volume) of the largest particle. Consequently, we can write

$$\alpha_2 = \int_0^R f(r)\, r dr, \quad \rho_2 = \int_0^R \rho_2^0 f(r)\, r dr,$$

where ρ_2^0 is the true density of the disperse phase. We set f(0) = f(R) = 0. It is postulated that there are sufficient particles of all sizes so that it can be assumed that the carrier phase and any given set of particles (whose sizes lie in the segment r', r'', where r' and r'' are any given values from the set [0, R]) are continua, filling exactly the same volume. The carrier phase is described by a model of a viscous liquid. Here, as the tensors of the surface forces σ_i^{kl} and the tensors of the viscous stresses τ_i^{kl}) we take [1, 3]

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